CS6200
Information Retrieval

PageRank Continued

with slides from
Hinrich Schütze and Christina Lioma

Exercise: Assumptions underlying PageRank

- Assumption 1: A link on the web is a quality signal – the
  author of the link thinks that the linked-to page is high-quality.
- Assumption 2: The anchor text describes the content of the linked-to page.
- Is assumption 1 true in general?
- Is assumption 2 true in general?

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  - Coordinated link creation by those who dislike the Church of Scientology
- Defused Google bombs: [dumb motherf…], [who is a failure?], [evil empire]

Origins of PageRank: Citation analysis (1)

- Citation analysis: analysis of citations in the scientific literature.
- Example citation: "Miller (2001) has shown that physical activity alters the metabolism of estrogens."
- We can view "Miller (2001)" as a hyperlink linking two scientific articles.
- One application of these "hyperlinks" in the scientific literature:
  - Measure the similarity of two articles by the overlap of other articles citing them.
  - This is called cocitation similarity.
  - Cocitation similarity on the web: Google's "find pages like this" or "Similar" feature.

Origins of PageRank: Citation analysis (2)

- Another application: Citation frequency can be used to measure the impact of an article .
  - Simplest measure: Each article gets one vote – not very accurate.
- On the web: citation frequency = inlink count
  - A high inlink count does not necessarily mean high quality ...
  - ... mainly because of link spam.
- Better measure: weighted citation frequency or citation rank
  - An article's vote is weighted according to its citation impact.
  - Circular? No: can be formalized in a well-defined way.

Origins of PageRank: Citation analysis (3)

- Better measure: weighted citation frequency or citation rank.
- This is basically PageRank.
- PageRank was invented in the context of citation analysis by Pinsker and Narin in the 1960s.
- Citation analysis is a big deal: The budget and salary of this lecturer are / will be determined by the impact of his publications!

Origins of PageRank: Summary

- We can use the same formal representation for
  - citations in the scientific literature
  - hyperlinks on the web
- Appropriately weighted citation frequency is an excellent measure of quality ...
  - ... both for web pages and for scientific publications.
- Next: PageRank algorithm for computing weighted citation frequency on the web.

Model behind PageRank: Random walk

- Imagine a web surfer doing a random walk on the web
  - Start at a random page
  - At each step, go out of the current page along one of the links on that page, equiprobably
- In the steady state, each page has a long-term visit rate.
- This long-term visit rate is the page's PageRank.
- PageRank = long-term visit rate = steady state probability.

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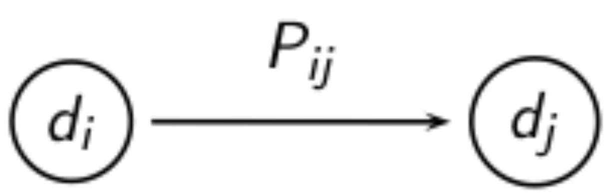
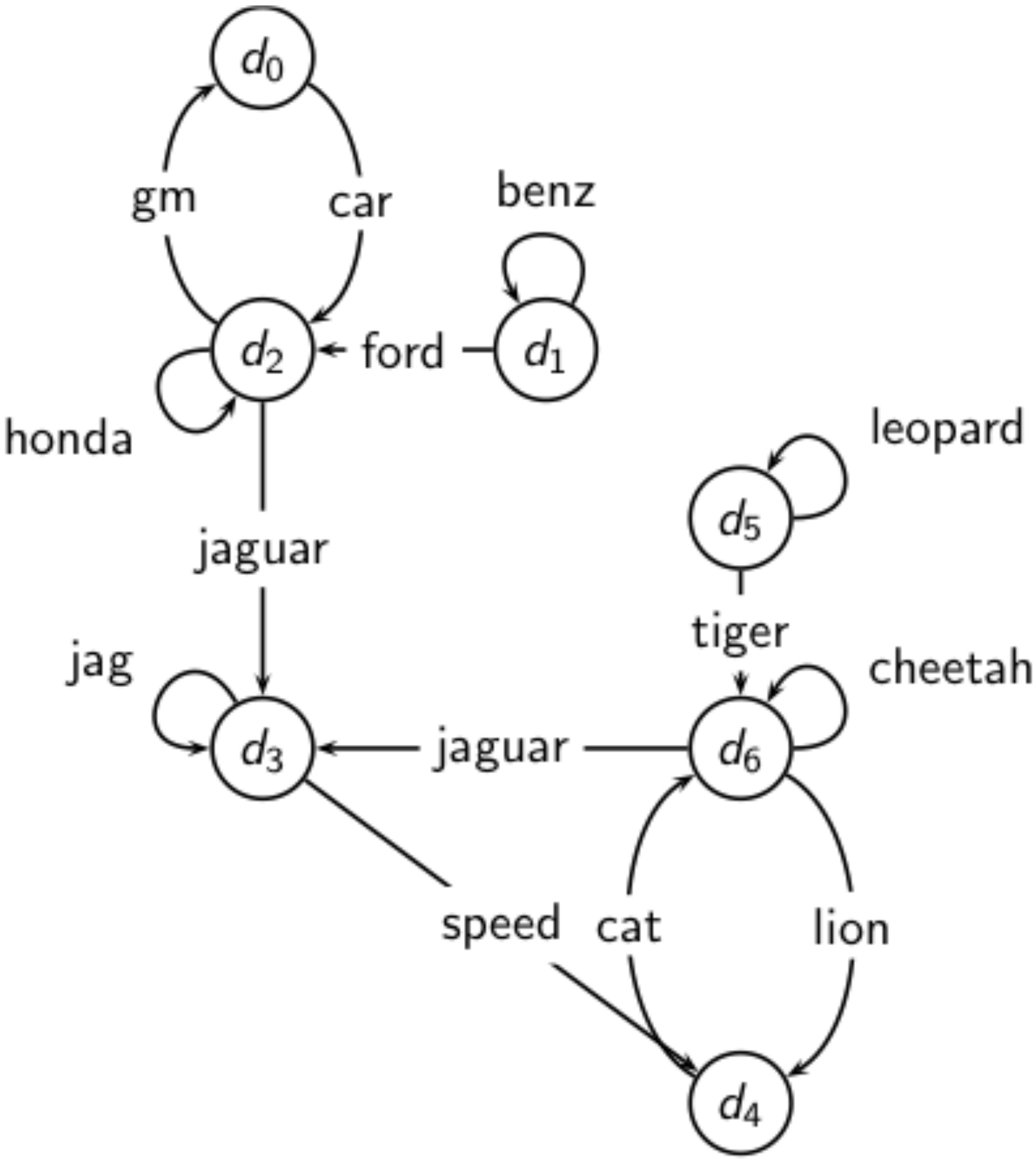
- A Markov chain consists of *N* states, plus an $N \times N$ transition probability matrix *P*.
- state = page
- At each step, we are on exactly one of the pages.
- For $1 \leq i, j \geq N$, the matrix entry P_{ij} tells us the probability of *j* being the next page, given we are currently on page *i*.
- Clearly, for all i, $\sum_{j=1}^{N} P_{ij} = 1$



Example web graph



Link matrix for example

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	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0	0	1	0	0	0	0
d_1	0	1	1	0	0	0	0
d_2	1	0	1	1	0	0	0
d_3	0	0	0	1	1	0	0
d_4	0	0	0	0	0	0	1
d_5	0	0	0	0	0	1	1
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d_1	0.00	0.50	0.50	0.00	0.00	0.00	0.00
d_2	0.33	0.00	0.33	0.33	0.00	0.00	0.00
d_3	0.00	0.00	0.00	0.50	0.50	0.00	0.00
d_4	0.00	0.00	0.00	0.00	0.00	0.00	1.00
d_5	0.00	0.00	0.00	0.00	0.00	0.50	0.50
d_6	0.00	0.00	0.00	0.33	0.33	0.00	0.33

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- The web graph must correspond to an ergodic Markov chain.
- First a special case: The web graph must not contain dead ends.

Dead ends

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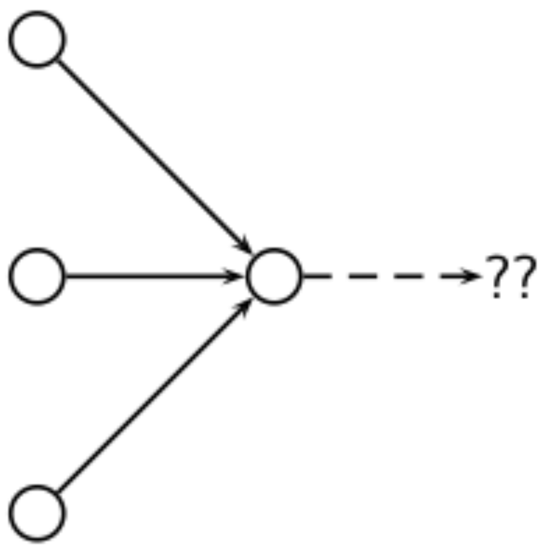

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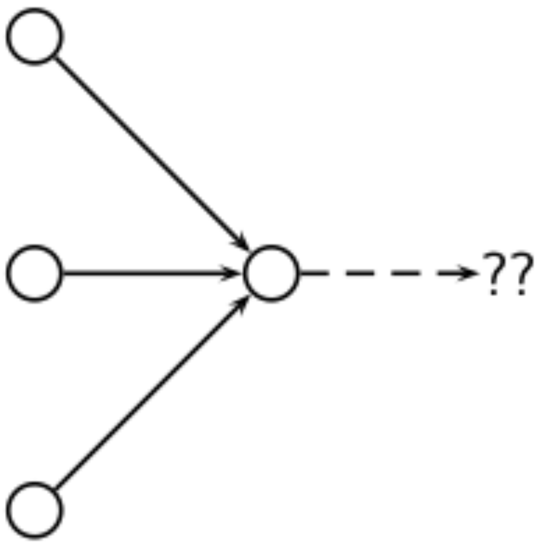

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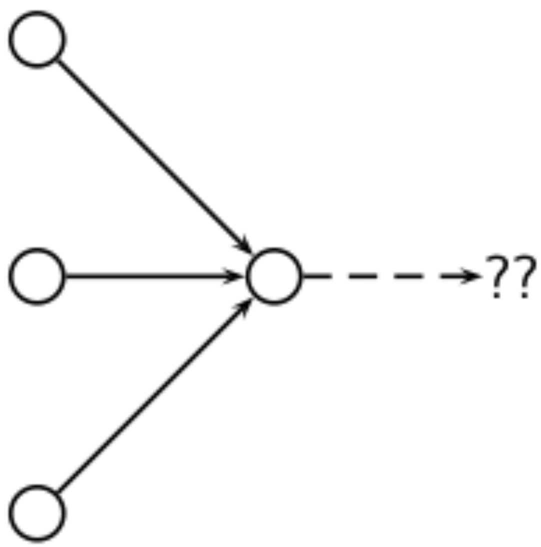

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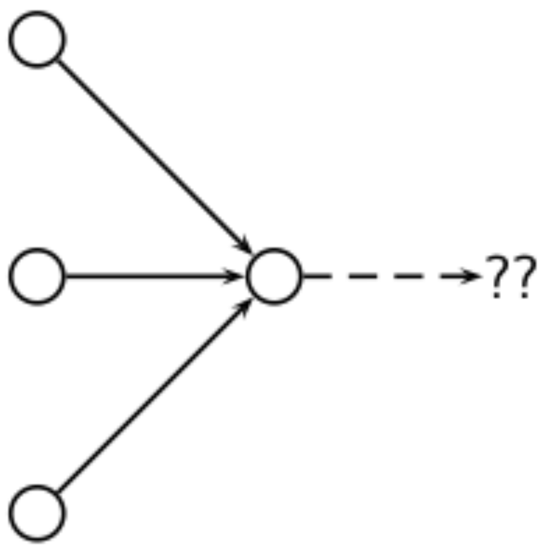

- The web is full of dead ends.
- Random walk can get stuck in dead ends.
- If there are dead ends, long-term visit rates are not well-defined (or non-sensical).

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- Note: "jumping" from dead end is independent of teleportation rate.

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- With teleporting, we cannot get stuck in a dead end.
- But even without dead ends in the original graph, we may not have well-defined long-term visit rates.
- More generally, we require that the Markov chain be ergodic.

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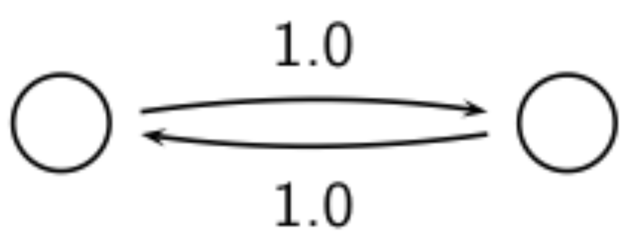
- A non-ergodic Markov chain:



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- $\Longrightarrow$ Web-graph+teleporting has a steady-state probability distribution.
- $\Longrightarrow$ Each page in the web-graph+teleporting has a PageRank.

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1	2	3	…	i	…	N-2	N-1	N	

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- $\Sigma x_i = 1$

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- If the probability vector is $\vec{x} = (x_1, \ldots, x_N)$, at this step, what is it at the next step?
- Recall that row i of the transition probability matrix P tells us where we go next from state i.
- So from $\vec{x}$, our next state is distributed as $\vec{x}P$.

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- π is the long-term visit rate (or PageRank) of page *i*.
- So we can think of PageRank as a very long vector – one entry per page.

Steady-state distribution: Example

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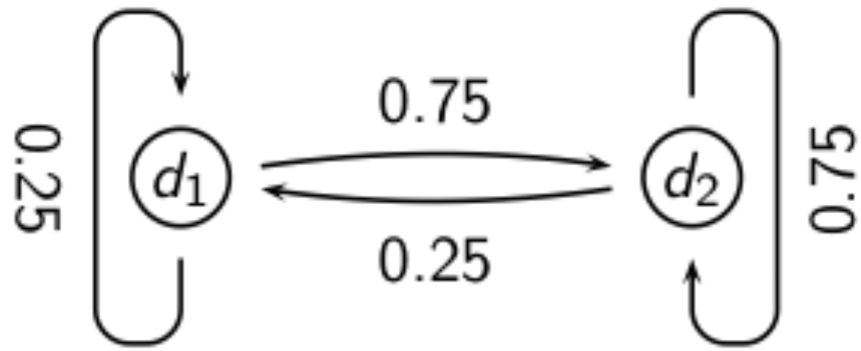
- What is the PageRank / steady state in this example?



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	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$		
			$P_{11} = 0.25$	$P_{12} = 0.75$
			$P_{21} = 0.25$	$P_{22} = 0.75$
t_0	0.25	0.75		
t_1				

PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.25, 0.75)$

$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$
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t_0	0.25	0.75	0.25	0.75
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- All transition probability matrices have largest eigenvalue

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- After k steps, we're at $\vec{x}P^k$.

One way of computing the PageRank $\vec{\pi}$

- Start with any distribution $\vec{x}$, e.g., uniform distribution
- After one step, we're at $\vec{x}P$.
- After two steps, we're at $\vec{x}P^2$.
- After k steps, we're at $\vec{x}P^k$.
- Algorithm: multiply $\vec{x}$ by increasing powers of P until convergence.

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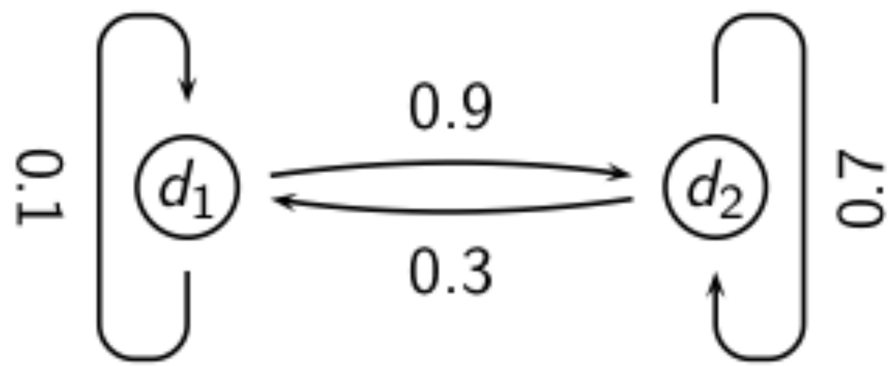
- After k steps, we're at $\vec{x}P^k$.
- Algorithm: multiply $\vec{x}$ by increasing powers of P until convergence.
- This is called the power method.
- Recall: regardless of where we start, we eventually reach the steady state π.
- Thus: we will eventually (in asymptotia) reach the steady state.

Power method: Example

Power method: Example

- What is the PageRank / steady state in this example?



Computing PageRank: Power Example

Computing PageRank: Power Example

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$			
			$P_{11} = 0.1$ $P_{21} = 0.3$	$P_{12} = 0.9$ $P_{22} = 0.7$	
t_0	*0*	*1*			$= \vec{x}P$
t_1					$= \vec{x}P^2$
t_2					$= \vec{x}P^3$
t_3					$= \vec{x}P^4$
					. . .
t_∞					$= \vec{x}P^\infty$

$$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$$
$$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$$

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t_0	*0*	*1*	*0.3*	*0.7*	$= \vec{x}P$
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t_0	*0*	*1*	*0.3*	*0.7*	$= \vec{x}P$
t_1	*0.3*	*0.7*			$= \vec{x}P^2$
t_2					$= \vec{x}P^3$
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Computing PageRank: Power Example

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			$P_{11} = 0.1$ $P_{21} = 0.3$	$P_{12} = 0.9$ $P_{22} = 0.7$	
t_0	0	1	0.3	0.7	$= \vec{x}P$
t_1	0.3	0.7	0.24	0.76	$= \vec{x}P^2$
t_2					$= \vec{x}P^3$
t_3					$= \vec{x}P^4$
					. . .
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t_0	*0*	*1*	*0.3*	*0.7*	$= \vec{x}P$
t_1	*0.3*	*0.7*	*0.24*	*0.76*	$= \vec{x}P^2$
t_2	*0.24*	*0.76*			$= \vec{x}P^3$
t_3					$= \vec{x}P^4$
					. . .
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t_0	0	1	0.3	0.7	$= \vec{x}P$
t_1	0.3	0.7	0.24	0.76	$= \vec{x}P^2$
t_2	0.24	0.76	0.252	0.748	$= \vec{x}P^3$
t_3					$= \vec{x}P^4$
					. . .
t_∞					$= \vec{x}P^\infty$

$$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$$
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t_2	*0.24*	*0.76*	*0.252*	*0.748*	$= \vec{x}P^3$
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			$P_{11} = 0.1$ $P_{21} = 0.3$	$P_{12} = 0.9$ $P_{22} = 0.7$	
t_0	*0*	*1*	*0.3*	*0.7*	$= \vec{x}P$
t_1	*0.3*	*0.7*	*0.24*	*0.76*	$= \vec{x}P^2$
t_2	*0.24*	*0.76*	*0.252*	*0.748*	$= \vec{x}P^3$
t_3	*0.252*	*0.748*	*0.2496*	*0.7504*	$= \vec{x}P^4$
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t_∞					$= \vec{x}P^\infty$

$$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$$
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t_0	0	1	0.3	0.7	$= \vec{x}P$
t_1	0.3	0.7	0.24	0.76	$= \vec{x}P^2$
t_2	0.24	0.76	0.252	0.748	$= \vec{x}P^3$
t_3	0.252	0.748	0.2496	0.7504	$= \vec{x}P^4$
			. . .		. . .
t_∞					$= \vec{x}P^\infty$

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t_0	*0*	*1*	*0.3*	*0.7*	$= \vec{x}P$
t_1	*0.3*	*0.7*	*0.24*	*0.76*	$= \vec{x}P^2$
t_2	*0.24*	*0.76*	*0.252*	*0.748*	$= \vec{x}P^3$
t_3	*0.252*	*0.748*	*0.2496*	*0.7504*	$= \vec{x}P^4$
				. . .	. . .
t_∞	*0.25*	*0.75*			$= \vec{x}P^\infty$

$$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$$
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Computing PageRank: Power Example

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$			
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t_0	*0*	*1*	*0.3*	*0.7*	$= \vec{x}P$
t_1	*0.3*	*0.7*	*0.24*	*0.76*	$= \vec{x}P^2$
t_2	*0.24*	*0.76*	*0.252*	*0.748*	$= \vec{x}P^3$
t_3	*0.252*	*0.748*	*0.2496*	*0.7504*	$= \vec{x}P^4$
			. . .		. . .
t_∞	*0.25*	*0.75*	*0.25*	*0.75*	$= \vec{x}P^\infty$

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t_0	*0*	*1*	*0.3*	*0.7*	$= \vec{x}P$
t_1	*0.3*	*0.7*	*0.24*	*0.76*	$= \vec{x}P^2$
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t_∞	*0.25*	*0.75*	*0.25*	*0.75*	$= \vec{x}P^\infty$

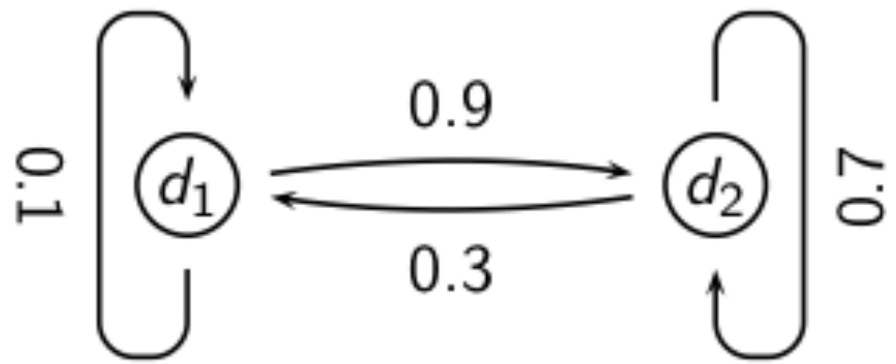
PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.25, 0.75)$

$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$

$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$

Power method: Example

- What is the PageRank / steady state in this example?



- The steady state distribution (= the PageRanks) in this example are 0.25 for d_1 and 0.75 for d_2.

Exercise: Compute PageRank using power method

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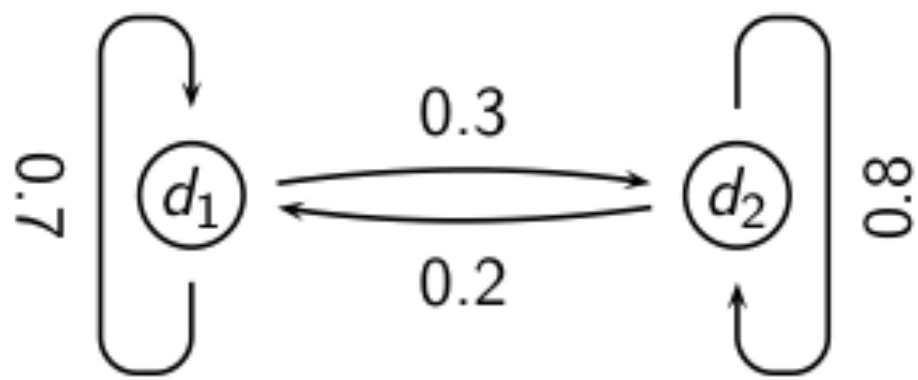

Solution

Solution

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	*0*	*1*		
t_1				
t_2				
t_3				
t_∞				

PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.4, 0.6)$

$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$

$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$

Solution

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$		
			$P_{11} = 0.7$ ***$P_{21} = 0.2$***	***$P_{12} = 0.3$*** ***$P_{22} = 0.8$***
t_0	***0***	***1***	***0.2***	***0.8***
t_1				
t_2				
t_3				
t_∞				

PageRank vector = $\vec{\pi} = (\pi_1, \pi_2) = (0.4, 0.6)$

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Solution

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			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	*0*	*1*	*0.2*	*0.8*
t_1	*0.2*	*0.8*		
t_2				
t_3				
t_∞				

PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.4, 0.6)$

$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$

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Solution

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$		
			$P_{11} = 0.7$	$P_{12} = 0.3$
			$P_{21} = 0.2$	$P_{22} = 0.8$
t_0	*0*	*1*	*0.2*	*0.8*
t_1	*0.2*	*0.8*	*0.3*	*0.7*
t_2				
t_3				
t_∞				

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$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$

$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$

Solution

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	*0*	*1*	***0.2***	***0.8***
t_1	***0.2***	***0.8***	***0.3***	***0.7***
t_2	***0.3***	***0.7***		
t_3				
t_∞				

PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.4, 0.6)$

$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$

$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$

Solution

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	***0***	***1***	***0.2***	***0.8***
t_1	***0.2***	***0.8***	***0.3***	***0.7***
t_2	***0.3***	***0.7***	***0.35***	***0.65***
t_3				
t_∞				

PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.4, 0.6)$

$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$

$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$

Solution

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	***0***	***1***	***0.2***	***0.8***
t_1	***0.2***	***0.8***	***0.3***	***0.7***
t_2	***0.3***	***0.7***	***0.35***	***0.65***
t_3	***0.35***	***0.65***		
t_∞				

PageRank vector = $\vec{\pi} = (\pi_1, \pi_2) = (0.4, 0.6)$

$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$

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Solution

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$		
			$P_{11} = 0.7$	$P_{12} = 0.3$
			$P_{21} = 0.2$	$P_{22} = 0.8$
t_0	*0*	*1*	*0.2*	*0.8*
t_1	*0.2*	*0.8*	*0.3*	*0.7*
t_2	*0.3*	*0.7*	*0.35*	*0.65*
t_3	*0.35*	*0.65*	*0.375*	*0.625*
t_∞				

PageRank vector = $\vec{\pi} = (\pi_1, \pi_2) = (0.4, 0.6)$

$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$

$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$

Solution

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	*0*	*1*	*0.2*	*0.8*
t_1	*0.2*	*0.8*	*0.3*	*0.7*
t_2	*0.3*	*0.7*	*0.35*	*0.65*
t_3	*0.35*	*0.65*	*0.375*	*0.625*
t_∞				

PageRank vector = $\vec{\pi} = (\pi_1, \pi_2) = (0.4, 0.6)$

$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$

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Solution

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	*0*	*1*	*0.2*	*0.8*
t_1	*0.2*	*0.8*	*0.3*	*0.7*
t_2	*0.3*	*0.7*	*0.35*	*0.65*
t_3	*0.35*	*0.65*	*0.375*	*0.625*
t_∞	*0.4*	*0.6*		

PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.4, 0.6)$

$$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$$

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Solution

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$		
			P_{11} = 0.7 ***P_{21} = 0.2***	***P_{12} = 0.3*** ***P_{22} = 0.8***
t_0	***0***	***1***	***0.2***	***0.8***
t_1	***0.2***	***0.8***	***0.3***	***0.7***
t_2	***0.3***	***0.7***	***0.35***	***0.65***
t_3	***0.35***	***0.65***	***0.375***	***0.625***
			. . .	
t_∞	***0.4***	***0.6***	***0.4***	***0.6***

PageRank vector = $\vec{\pi} = (\pi_1, \pi_2) = (0.4, 0.6)$

$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$

$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$

PageRank summary

PageRank summary

- Preprocessing

PageRank summary

- Preprocessing
  - Given graph of links, build matrix P

PageRank summary

- Preprocessing
  - Given graph of links, build matrix P
  - Apply teleportation

PageRank summary

- Preprocessing
  - Given graph of links, build matrix *P*
  - Apply teleportation
  - From modified matrix, compute $\vec{\pi}$

PageRank summary

- Preprocessing
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  - Return reranked list to the user

PageRank issues

- Real surfers are not random surfers.
  - Examples of nonrandom surfing: back button, short vs. long paths, bookmarks, directories – and search!
  - → Markov model is not a good model of surfing.
  - But it's good enough as a model for our purposes.
- Simple PageRank ranking (as described on previous slide) produces bad results for many pages.
  - Consider the query [video service].
  - The Yahoo home page (i) has a very high PageRank and (ii) contains both *video* and *service*.
  - If we rank all pages containing the query terms according to PageRank, then the Yahoo home page would be top-ranked.
  - Clearly not desirable.

How important is PageRank?

- Frequent claim: PageRank is the most important component of web ranking.
- The reality:
  - There are several components that are at least as important: e.g., anchor text, phrases, proximity, tiered indexes ...
  - Rumor has it that PageRank in his original form (as presented here) now has a negligible impact on ranking!
  - However, variants of a page's PageRank are still an essential part of ranking.
  - Addressing link spam is difficult and crucial.